Math 10

Lesson 2-3 Factoring trinomials

# Lesson Objectives:

a) To see the patterns in multiplying binomials that can be used to factor trinomials into binomials.

b) To factor trinomials of the form *a*x2 + *b*x + *c*.

# Binomial multiplication – hunting for patterns

In the previous lesson we saw how the distributive property could be used to multiply binomials together. In this lesson we are interested in doing the reverse – **we want to factor trinomials into binomials**. Perhaps if we studied how binomials multiply together we can find some patterns that may help us to reverse the process when we factor a trinomial. With this in mind, let’s do a few more binomial multiplications to see if any pattern(s) become evident. Consider the following five binomial multiplications:

(x + 3)(x + 2)

= x(x + 2) + 3(x + 2)

= x2 + 2x + 3x + 6

= x2 + 5x + 6

(x ─ 3)(x + 2)

= x(x + 2) ─ 3(x + 2)

= x2 + 2x ─ 3x ─ 6

= x2 ─ x ─ 6

(2x ─ 3)(x + 2)

= 2x(x + 2) ─ 3(x + 2)

= 2x2 + 4x ─ 3x ─ 6

= 2x2 + x ─ 6

(x + 3)(x ─ 3)

= x(x ─ 3) + 3(x ─ 3)

= x2 ─ 3x + 3x ─ 9

= x2 ─ 9

(2x ─ 3)(5x ─ 2)

= 2x(5x ─ 2) ─ 3(5x ─ 2)

= 10x2 ─ 4x ─ 15x + 6

= 2x2 ─ 19x + 6

The first pattern is that the **first two terms of the binomials multiply together to form the first term of each trinomial**.

(**x** + 3)(**x** + 2)

= x(x + 2) + 3(x + 2)

= x2 + 2x + 3x + 6

= **x2** + 5x + 6

(**x** ─ 3)(**x** + 2)

= x(x + 2) ─ 3(x + 2)

= x2 + 2x ─ 3x ─ 6

= **x2** ─ x ─ 6

(**2x** ─ 3)(**x** + 2)

= 2x(x + 2) ─ 3(x + 2)

= 2x2 + 4x ─ 3x ─ 6

= **2x2** + x ─ 6

(**x** + 3)(**x** ─ 3)

= x(x ─ 3) + 3(x ─ 3)

= x2 ─ 3x + 3x ─ 9

= **x2** ─ 9

(**2x** ─ 3)(**5x** ─ 2)

= 2x(5x ─ 2) ─ 3(5x ─ 2)

= 10x2 ─ 4x ─ 15x + 6

= **10x2** ─ 19x + 6

The second pattern is that the **last two terms of the binomials multiply together to form the last term of each trinomial**.

(x **+ 3**)(x **+ 2**)

= x(x + 2) + 3(x + 2)

= x2 + 2x + 3x + 6

= x2 + 5x **+ 6**

(x **– 3**)(x **+ 2**)

= x(x + 2) ─ 3(x + 2)

= x2 + 2x ─ 3x ─ 6

= x2 ─ x **– 6**

(2x **– 3**)(x **+ 2**)

= 2x(x + 2) ─ 3(x + 2)

= 2x2 + 4x ─ 3x ─ 6

= 2x2 + x **– 6**

(x **+ 3**)(x **– 3**)

= x(x – 3) + 3(x – 3)

= x2 – 3x + 3x – 9

= x2 **– 9**

(2x **– 3**)(5x **– 2**)

= 2x(5x ─ 2) ─ 3(5x ─ 2)

= 10x2 ─ 4x ─ 15x + 6

= 2x2 ─ 19x **+ 6**

The third pattern is that the **middle term of the trinomial is formed when we add the x-terms** **together**.

(x + 3)(x + 2)

= x(x + 2) + 3(x + 2)

= x2 **+ 2x + 3x** + 6

= x2 **+ 5x** + 6

(x ─ 3)(x + 2)

= x(x + 2) ─ 3(x + 2)

= x2 **+ 2x ─ 3x** ─ 6

= x2 **─ x** ─ 6

(2x ─ 3)(x + 2)

= 2x(x + 2) ─ 3(x + 2)

= 2x2 **+ 4x ─ 3x** ─ 6

= 2x2 **+ x** ─ 6

(x + 3)(x ─ 3)

= x(x ─ 3) + 3(x ─ 3)

= x2 **─ 3x + 3x** ─ 9

= x2 + **0x** ─ 9

(2x ─ 3)(5x ─ 2)

= 2x(5x ─ 2) ─ 3(5x ─ 2)

= 10x2 **─ 4x ─ 15x** + 6

= 2x2 **─ 19x** + 6

The fourth pattern is a little harder to see, but it leads directly to something we can use. The pattern can be seen in the third line of each binomial multiplication. When we **multiply the coefficients of the middle terms** and we **multiply the end term coefficients**, we get the **same number**!!

(x + 3)(x + 2)

= x(x + 2) + 3(x + 2)

= 1x2 **+ 2**x **+ 3**x + 6

= x2 + 5x + 6

(x ─ 3)(x + 2)

= x(x + 2) ─ 3(x + 2)

= 1x2 **+ 2**x **─ 3**x ─ 6

= x2 ─ x ─ 6

(2x ─ 3)(x + 2)

= 2x(x + 2) ─ 3(x + 2)

= 2x2 **+ 4**x **─ 3**x ─ 6

= 2x2 + x ─ 6

(x + 3)(x ─ 3)

= x(x ─ 3) + 3(x ─ 3)

= 1x2 **─ 3**x **+ 3**x ─ 9

= x2 ─ 9

(2x ─ 3)(5x ─ 2)

= 2x(5x ─ 2) ─ 3(5x ─ 2)

= 10x2 **─ 4**x **─ 15**x + 6

= 2x2 ─ 19x + 6

**2**·**3** = 6

1·6 = 6

**2**·–**3** = –6

1·–6 = –6

**4**·–**3** = –12

2·–6 = –12

**–4**·**–15** = 60

10·6 = 60

**–3**·**3** = –9

1·–9 = –9

Ah ha!! or, as Archimedes would have said, “Eureka!!”. When we multiply the end terms of a trinomial together and then write its factors, two of the factors add to form the middle term coefficient. Thus we have a pattern that we can use to factor a trinomial:

To factor any trinomial of the form *a*x2 + *b*x + *c*, decompose *b*x into two terms whose coefficients have a sum of *b* and a product equal to *a·c*.

(That is one tough sentence to interpret!!) The idea becomes simpler when we break it down into a few steps and then show some examples.

The basic steps for factoring trinomials with the form *a*x2 + *b*x + *c*, are:

1. Multiply *a·c* to produce the number.

**If this cannot be found, the trinomial cannot be factored by this method.**

1. List the factors of the number.
2. Find two factors of the number that add up to *b*.
3. Decompose *bx* into the two factors.

The word **composition** means “to bring together.” Therefore, to **decompose** something is to split it apart.

1. Factor the polynomial by grouping.

# Factoring trinomials – examples

The basic steps are reproduced below so you do not have to flip pages back and forth.

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1. Factor the polynomial by grouping.

Let’s look at several examples to get the idea.

**Example 1** Factor x2 + 5x + 4 using (a) decomposition and (b) algebra tiles

a)

a = 1 and c = 4 ∴ a·c = 4

Find two integers with a product of 4 and a sum of 5

**The product is positive and the sum is positive. What signs do the integers need to have?**

|  |  |
| --- | --- |
| Factors of 4 | Sum |
| 1 x 4 | 5 |
| 2 x 2 | 4 |

Decompose the middle term into its factors (1 & 4).

x2 + 5x + 4

= x2 + 4x + x + 4

Factor by grouping the first two terms and the last two terms.

=(x2 + 4x) + (x + 4)

= x(x + 4) + 1(x + 4)

= (x + 1)(x + 4)

b)

Arrange one x2-tile, five x-tiles and four 1-tiles into a rectangle. Then place tiles around the rectangle to show its dimensions.

The dimensions of the rectangle are x + 4 and x + 1.

∴ x2 + 5x + 4 = (x + 4)(x + 1)

**Example 2** Factor 3x2 + 8x + 4 using (a) decomposition and (b) algebra tiles

When there are coefficients other than 1, it is wise to check if there is a common factor that can be removed. In this case there is no common factor for 3, 8 and 4.

**The product is positive and the sum is positive. ∴ both factors will be positive**

a)

a·c = 3·4 = 12

Find two integers with a product of 4 and a sum of 5

|  |  |
| --- | --- |
| Factors of 4 | Sum |
| 1 x 12 | 13 |
| 2 x 6 | 8 |
| 3 x 4 | 12 |

Decompose the middle term into its factors (2 & 6).

3x2 + 8x + 4

= 3x2 + 2x + 6x + 4

Factor by grouping the first two terms and the last two terms.

=(3x2 + 2x) + (6x + 4)

= x(3x + 2) + 2(3x + 2)

= (x + 2)(3x + 2)

b)

Arrange three x2-tiles, eight x-tiles and four 1-tiles into a rectangle. Then place tiles around the rectangle to show its dimensions.

The dimensions of the rectangle are 3x + 2 and x + 2.

Check

(3x + 2)(x + 2)

= 3x(x + 2) + 2(x + 2)

= 3x2 + 6x +2x + 4

= 3x2 + 8x + 4

**Example 3** Factor 24x2 – 30x – 9

A check for the greatest common factor of 24, 30 and 9 reveals 3 as the GCF.

 24x2 – 30x – 9

**The product is negative and the sum is negative. ∴ one factor will be negative**

**We note that the sum is correct accept for sign.**

**Reverse the signs on the factors.**

a·c = 8· –3 = –24

Find two integers with a product of 24 and a sum of –10

|  |  |
| --- | --- |
| Factors of –24 | Sum |
| –1 x 24 | 23 |
| ­–2 x 12 | 10 |
| ­2 x –12 | –10 |

= 3(8x2 – 10x – 3)

= 3(8x2 – 12x + 2x – 3)

= 3[(8x2 – 12x) + (2x – 3)]

= 3[4x(2x – 3) + 1(2x – 3)]

= **3(4x + 1)(2x – 3)**

**Question 1**

If possible, factor each trinomial.

a) x2 + 5x + 6 b) x2 – 29x + 28 c) x2 – 3xy – 18y2

**Question 2**

If possible, factor each trinomial

a) 2x2 + 7x – 4 b) –3s2 – 51s – 30 c) 3x2 + x – 4

**Question 3**

If possible, factor each trinomial

a) x2 + 7x + 10 b) 6x2 – 5xy + y2 c) 2y2 + 7xy + 3x2

# Assignment

1. Write the trinomial represented by each rectangle of algebra tiles. Then, determine the dimensions of each rectangle.



2. Factor each trinomial.

a) 2x2 + 5x + 3

b) 3x2 + 7x + 4

c) 3x2 + 7x – 6

d) 6x2 + 11x + 4

3. Factor, if possible.

a) x2 + 7x + 10 b) j2 + 12j + 27

c) k2 + 5k + 4 d) p2 + 9p + 12

e) d2 + 10d + 24 f) c2 + 4cd + 21d2

4. Factor each trinomial.

a) m2 – 7m + 10 b) s2 + 3s – 10

c) f2 – 7f + 6 d) g2 – 5g – 14

e) b2 – 3b – 4 f) 2r2 – 14rs + 24s2

5. Factor, if possible.

a) 2x2 + 7x + 5 b) 6y2 + 19y + 8

c) 3m2 + 10m + 8 d) 10w2 + 15w + 3

e) 12q2 + 17q + 6 f) 3x2 + 7xy + 2y2

6. Factor, if possible.

a) 4x2 – 11x + 6 b) w2 + 11w + 25

c) x2 – 5x + 6 d) 2m2 + 3m – 9

e) 6x2 – 3xy – 3y2 f) 12y2 + y – 1

g) 6c2 + 7cd – 10d2 h) 4k2 + 15k + 9

i) a2 + 11ab + 24b2 j) 6m2 + 13mn + 2n2

7. Identify binomials that represent the length and width of each rectangle. Then, calculate the dimensions of the rectangle if x = 15 cm.



8. You can estimate the height, h, in metres, of a toy rocket at any time, t, in seconds, during its flight. Use the formula h = –5t2 + 23t + 10. Write the formula in factored form. Then, calculate the height of the rocket 3 s after it is launched.

9. You have been asked to factor the expression 30x2 – 39xy – 9y2. What are the factors?

10. A rescue worker launches a signal flare into the air from the side of a mountain. The height of the flare can be represented by the formula *h* = –16*t*2 + 144*t* + 160. In the formula, *h* is the height, in feet, above ground, and *t* is the time, in seconds.

a) What is the factored form of the formula?

b) What is the height of the flare after 5.6 s?